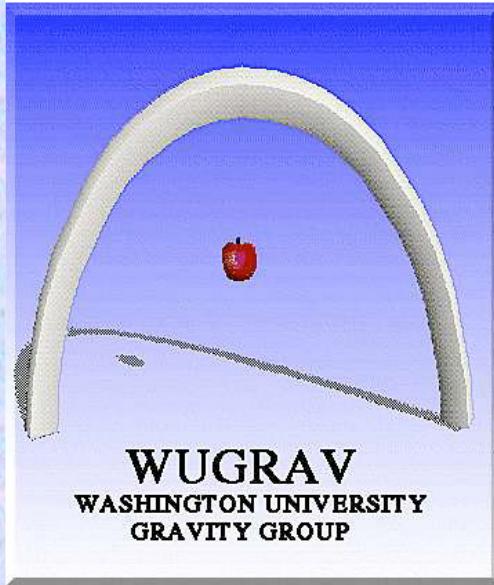


On the unreasonable effectiveness of post-Newtonian theory in gravitational-wave physics



Clifford Will
Washington University, St. Louis

FermiLab Colloquium, 9 Jan 2008

Interferometers Around The World

LIGO Hanford 4&2 km



GEO Hannover 600 m



LIGO Livingston 4 km

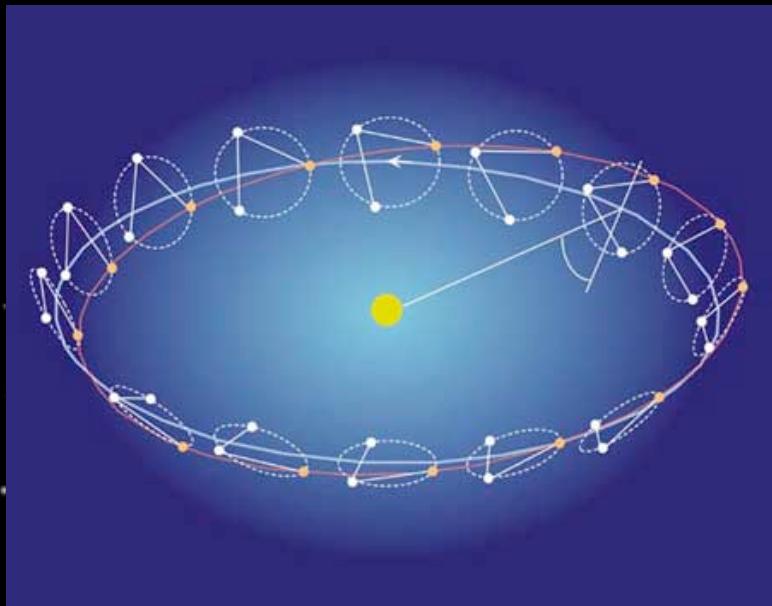
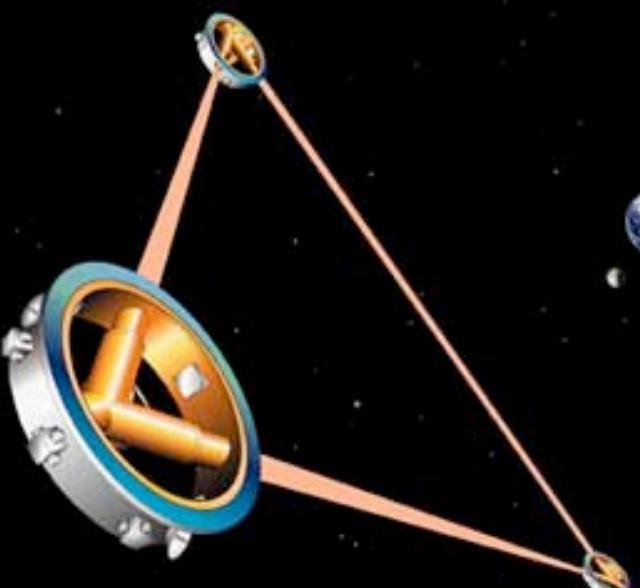
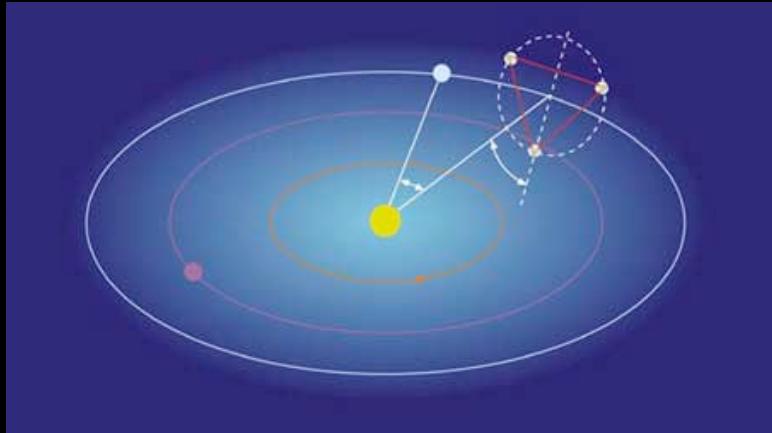
Virgo Cascina 3 km



TAMA Tokyo
300 m



LISA: a space interferometer for 2018



Inspiralling Compact Binaries - The Workhorse Source

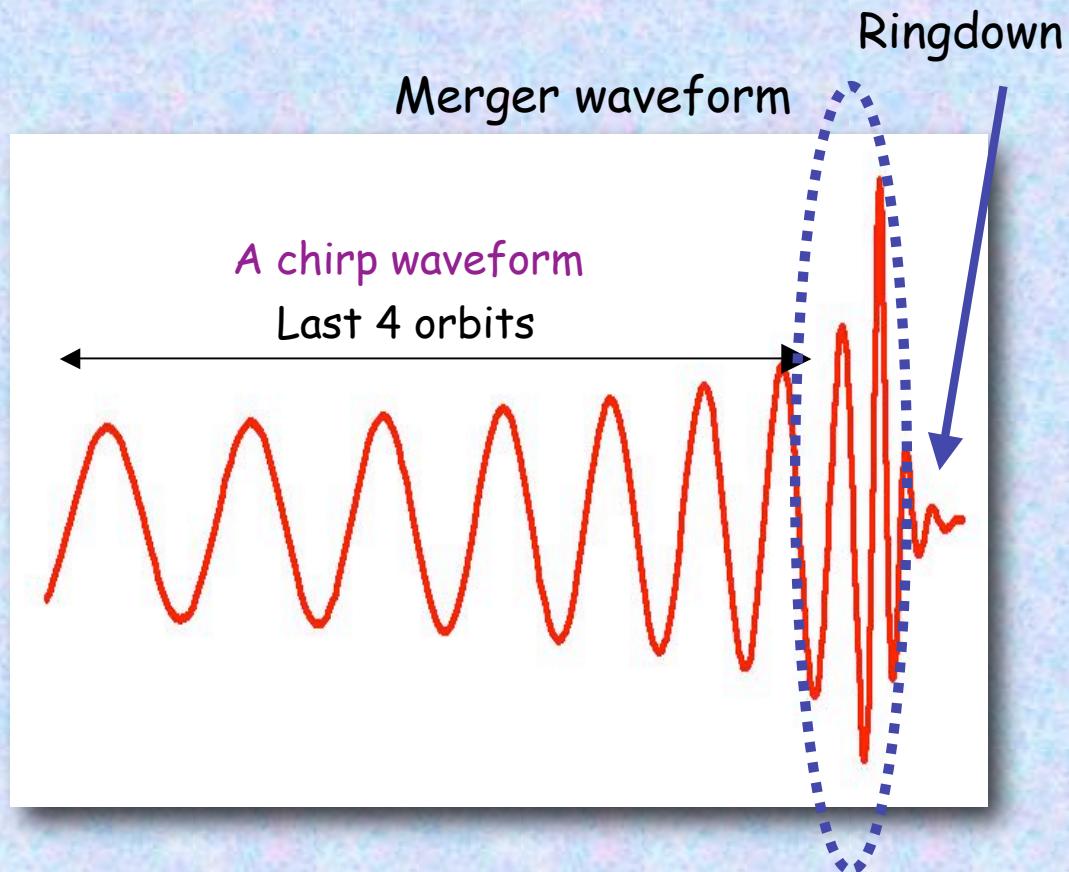
- Fate of the binary pulsar in 100 My
- GW energy loss drives pair toward merger

LIGO-VIRGO

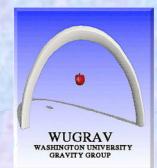
- Last few minutes (10K cycles) for NS-NS
- 40 - 700 per year by 2014
- BH inspirals could be more numerous

LISA

- MBH pairs(10^5 - $10^7 M_s$) in galaxies to large Z
- EMRIs



Extremely accurate theoretical templates needed to maximize detection and parameter estimation



On the unreasonable effectiveness of post-Newtonian theory in gravitational-wave physics

The unreasonable effectiveness of mathematics in the
natural sciences

By Eugene Wigner

Communications in Pure and Applied Mathematics

Vol. 13, No. I (February 1960).

*“....the enormous usefulness of mathematics in the natural sciences
is something bordering on the mysterious and ... there is no rational
explanation for it.”*

On the unreasonable effectiveness of post-Newtonian theory in gravitational-wave physics

- Introduction
- The problem of motion & radiation - a history
- Post-Newtonian theory
- “Unreasonable accuracy”
 - Binary pulsars
 - Gravitational-wave kicks
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Clifford Will, FermiLab Colloquium, 9 Jan 2008

The post-Newtonian approximation

$$\varepsilon \sim (v/c)^2 \sim (Gm/rc^2) \sim (p/\rho c^2)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon h^{(1)}{}_{\mu\nu} + \varepsilon^2 h^{(2)}{}_{\mu\nu} + \dots$$

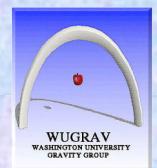
$$G_{\mu\nu} = 8\pi T_{\mu\nu} \quad (G = c = 1)$$

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(u_\mu u_\nu + g_{\mu\nu})$$



The problem of motion & radiation

- Geodesic motion
- 1916 - Einstein - gravitational radiation (wrong by factor 2)
- 1916 - Droste & De Sitter - n-body equations of motion
- 1918 - Lense & Thirring - motion in field of spinning body
- 1937-38 - center-of-mass acceleration (Levi-Civita/Eddington-Clark)
- 1938 - EH paper
- 1960s - Fock & Chandrasekhar - PN approximation
- 1967 - the Nordtvedt effect
- 1974 - numerical relativity - BH head-on collision
- 1974 - discovery of PSR 1913+16
- 1976 - Ehlers et al - critique of foundations of EOM
- 1976 - PN corrections to gravitational waves (EWW)
- 1979 - measurement of damping of binary pulsar orbit
- 1990s to now - EOM and gravitational waves to HIGH PN order
 - driven by requirements for GW detectors
 - $(v/c)^{12}$ beyond Newtonian gravity
 - interface with numerical relativity



DIRE: Direct integration of the relaxed Einstein equations

Einstein's Equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

"Relaxed" Einstein's Equations

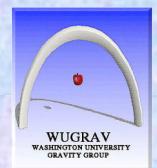
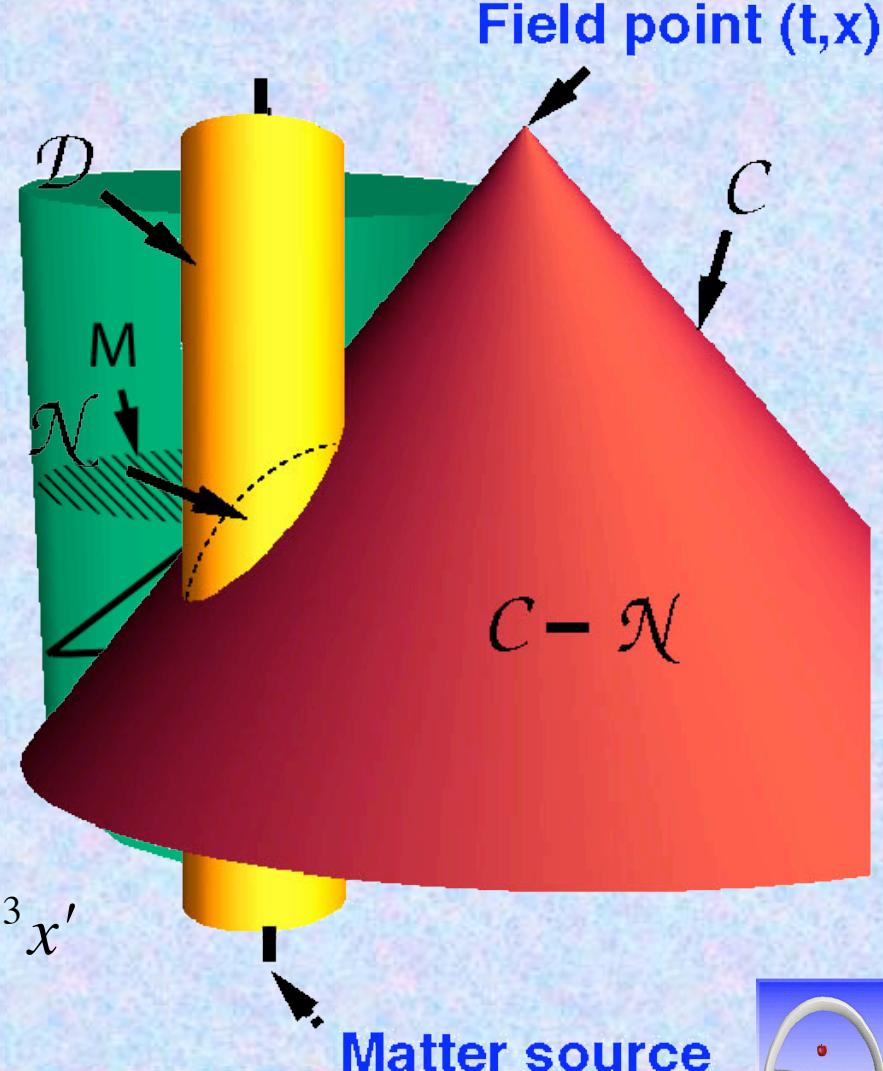
$$h^{\mu\nu} \equiv \eta^{\mu\nu} - \sqrt{-g} g^{\mu\nu}$$

$$\partial_\nu h^{\mu\nu} = 0$$

$$\square h^{\mu\nu} = -16\pi(-g)(T^{\mu\nu} + t^{\mu\nu})$$

$$h^{\mu\nu} = 4 \int_C \frac{\tau^{\mu\nu}(t - |x - x'|, x')}{|x - x'|} d^3x'$$

$$\nabla_\nu T^{\mu\nu} = 0, \text{ or } \partial_\nu \tau^{\mu\nu} = 0$$



PN equations of motion for compact binaries

$$\vec{a} = -\frac{m}{r^3} \vec{x} + 1PN + 1PN_{SO} + 1PN_{SS} + 2PN + 2.5PN$$

$$+ 3PN$$

B F S

$$+ 3.5PN$$

W B

$$+ 3.5PN_{SO}$$

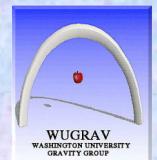
W

$$+ 3.5PN_{SS}$$

W

B = Blanchet, Damour, Iyer et al
F = Futamase, Itoh
S = Schäfer, Jaranowski
W = WUGRAV

WUGRAV contributions by graduate students Mike Pati,
Tom Mitchell, Han Wang, Jing Zeng



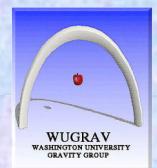
Gravitational energy flux for compact binaries

$$\dot{E} = \dot{E}_{quad} + 1PN \quad \text{Wagoner \& CW 76}$$

$$+ 1PN_{SO} + 1PN_{SS} \quad \text{W}$$

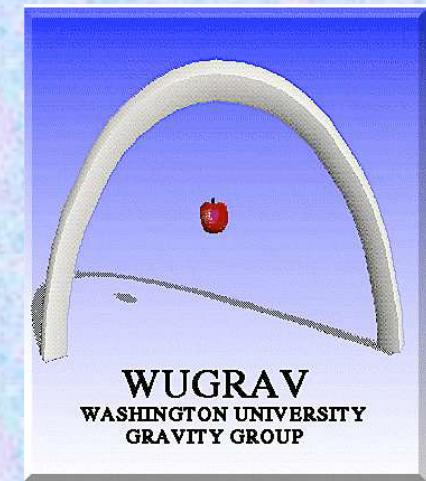
$$\begin{aligned} \mathcal{L} = \frac{32c^5}{5G}\nu^2x^5 & \left\{ 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu \right)x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2 \right)x^2 \right. \\ & + \left(-\frac{8191}{672} - \frac{583}{24}\nu \right)\pi x^{5/2} \\ & + \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16x) \right. \\ & \quad \left. + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2 \right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3 \right]x^3 \\ & \left. + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2 \right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right) \right\}. \end{aligned}$$

$$+ 3.5PN \quad \text{B}$$



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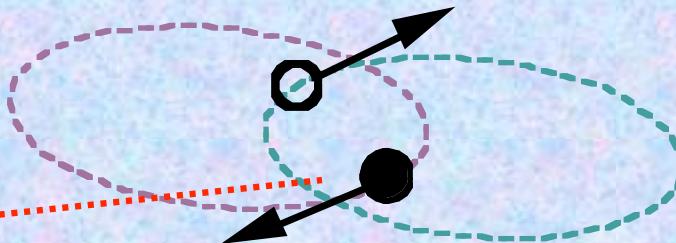
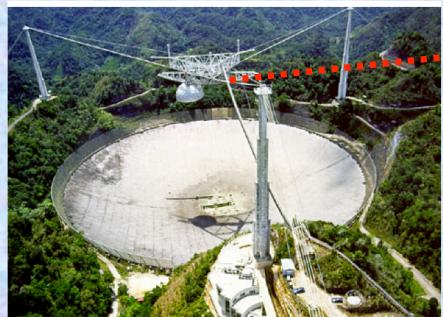
Clifford Will, FermiLab Colloquium, 9 Jan 2008

The Binary Pulsar: Is strong gravity "effaced"?

Discovery: 1974

Pulse period: 59 ms (16cps)

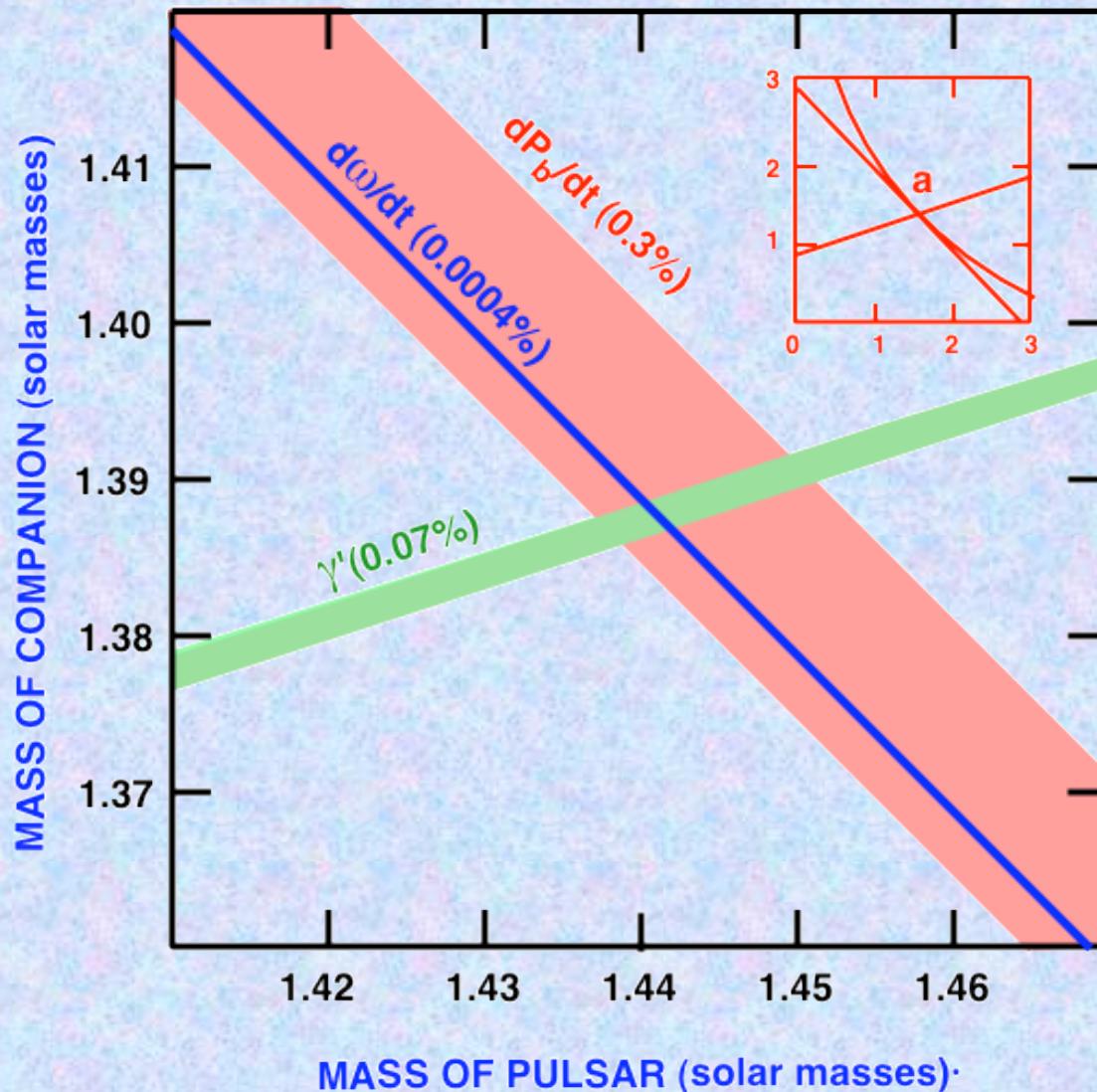
Orbit period: 8 hours



Parameter	Value
Keplerian	
Pulse Period (ms)	59.029997929613(7)
Orbit Period (days)	0.322997448930(4)
Eccentricity	0.6171338(4)
Post-Keplerian	
Periastron Shift ($d\omega/dt$ °/yr)	4.226595(5)
Pulsar Clock Shifts (ms)	4.2919(8)
Orbit Decay (dP_b/dt 10^{-12})	-2.4184(9)



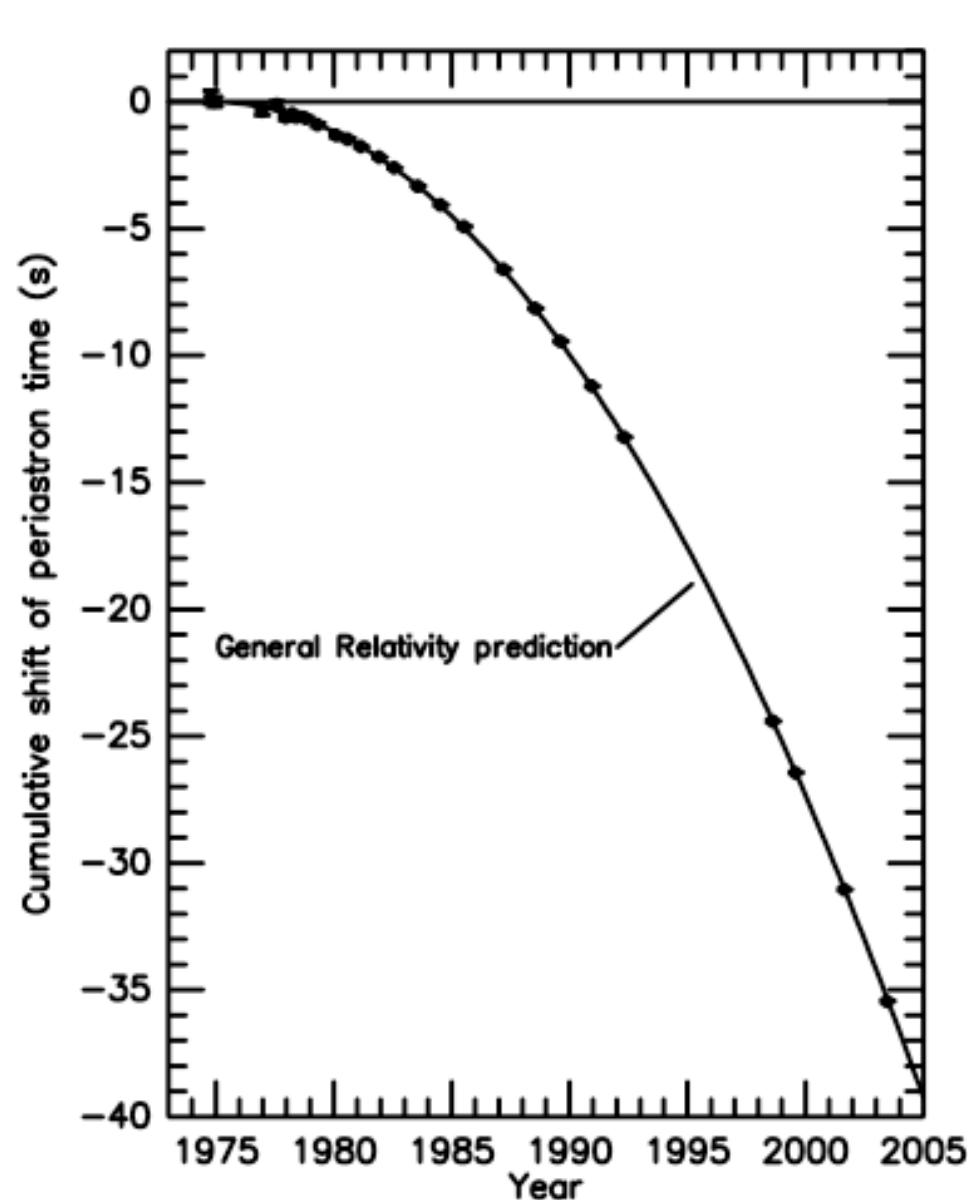
PSR 1913+16: Concordance with GR



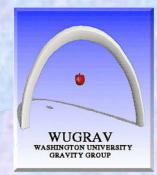
$$m_p = 1.4411(7) M_{\text{sun}} \quad m_c = 1.3874(7) M_{\text{sun}}$$



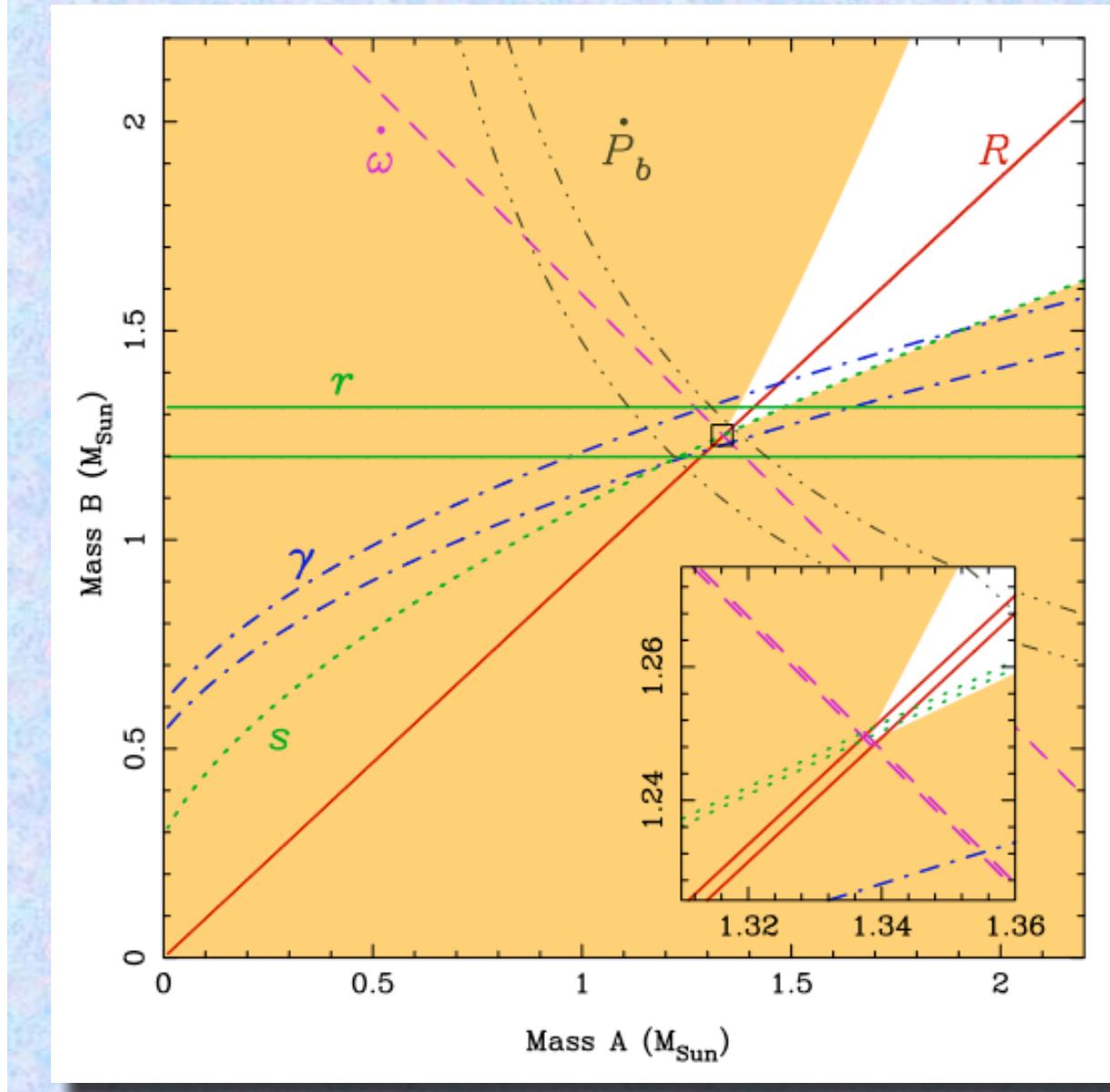
Decay of the orbit of PSR 1913+16



From Weisberg
& Taylor
(astro-ph/0407149)

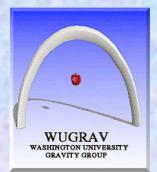


The Double Pulsar



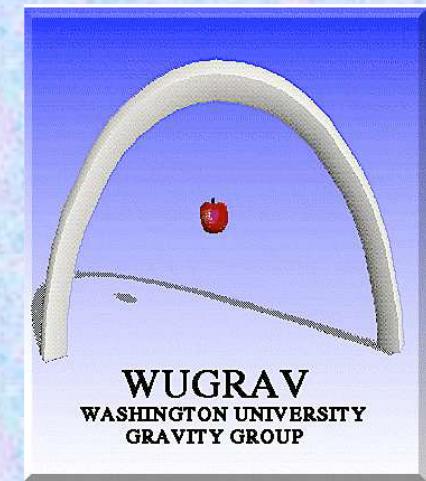
J 0737-3039

- 0.10 day orbit
- two pulsars seen!
- $d\omega/dt = 17^\circ/\text{yr}$
- $\sin i = 0.9995$
- $dP_b/dt \sim 6\%$



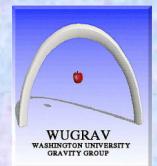
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Clifford Will, FermiLab Colloquium, 9 Jan 2008

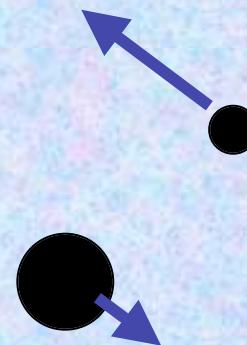
Numerical Relativity meets PN Theory (NRm3PN)



Radiation of momentum and the recoil of massive black holes

General Relativity

- Interference between quadrupole and higher moments
Peres (62), Bonnor & Rotenberg (61), Papapetrou (61), Thorne (80)
- "Newtonian effect" for binaries
Fitchett (83), Fitchett & Detweiler (84)
- 1 PN correction term
Wiseman (92)



Astrophysics

- MBH formation by mergers is affected if BH ejected from early galaxies
- Ejection from dwarf galaxies or globular clusters
- Displacement from center could affect galactic core
Merritt, Milosavljevic, Favata, Hughes & Holz (04)
Favata, Hughes & Holz (04)



How black holes get their kicks

Favata, et al

Getting a kick out of numerical relativity

Centrella, et al

Total recoil: the maximum kick...

Gonzalez et al

A swift kick in the astrophysical compact object

Boot, Foot, et al

Recoiling from numerical relativists

Curly, Larry & Moe

Radiation of momentum to 2PN order

Blanchet, Qusailah & CW (2005)

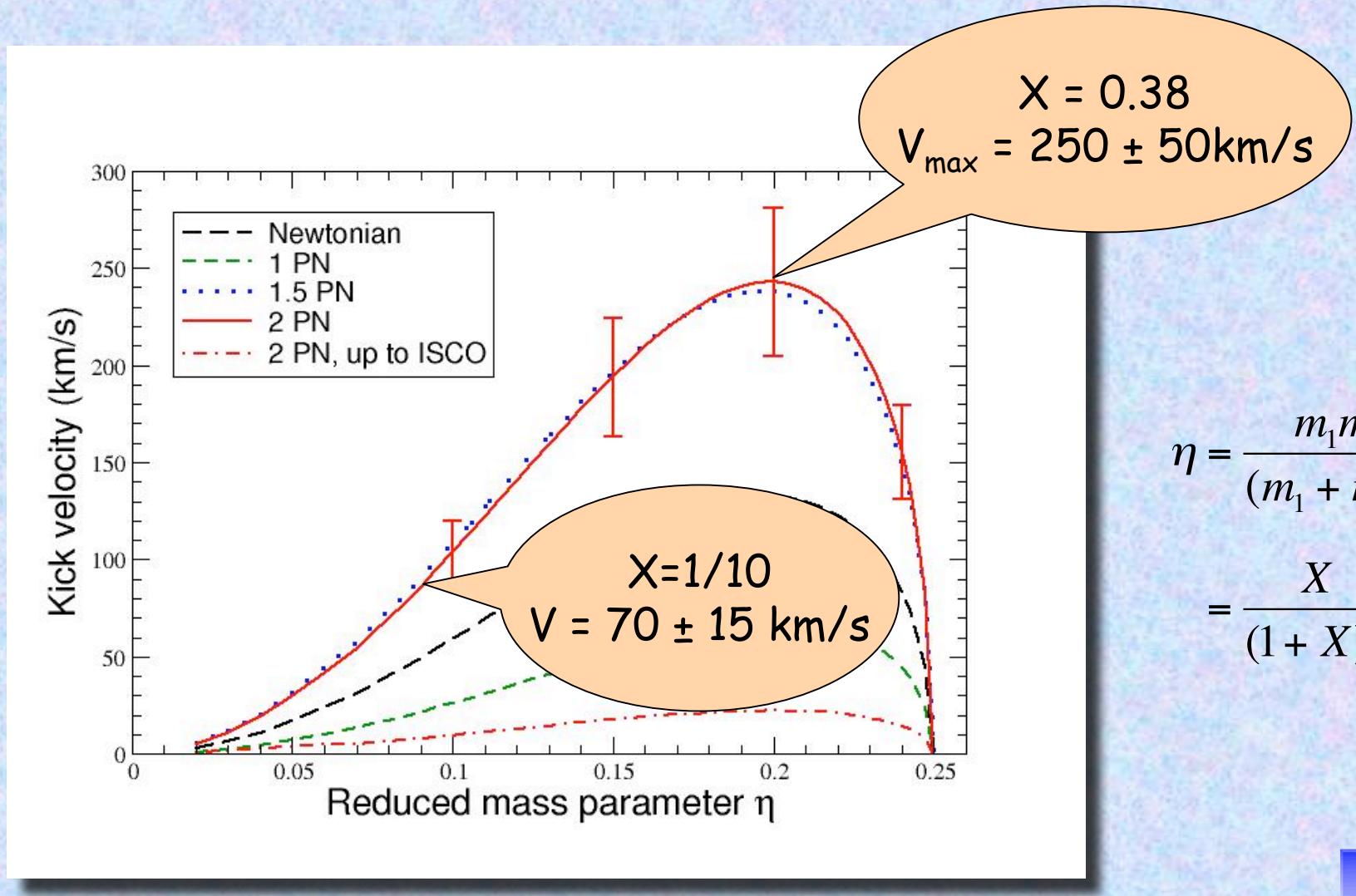
- Calculate relevant multipole moments to 2PN order
quadrupole, octupole, current quadrupole, etc
- Calculate momentum flux for quasi-circular orbit [$x=(m\omega)^{2/3} \approx (v/c)^2$]
recoil = -flux

$$\frac{d\vec{P}}{dt} = -\frac{464}{105} \frac{\delta m}{m} \eta^2 x^{11/2} \left[1 + \left(-\frac{452}{87} - \frac{1139}{522} \eta \right) x + \frac{309}{58} \pi x^{3/2} \right. \\ \left. + \left(-\frac{71345}{22968} + \frac{36761}{2088} \eta + \frac{147101}{68904} \eta^2 \right) x^2 \right] \vec{\lambda}$$

- Integrate up to ISCO (6m) for adiabatic inspiral
- Match quasicircular orbit at ISCO to plunge orbit in Schwarzschild
- Integrate with respect to "proper ω " to horizon ($x \rightarrow 0$)



Recoil velocity as a function of mass ratio

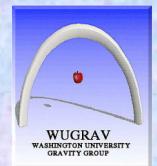
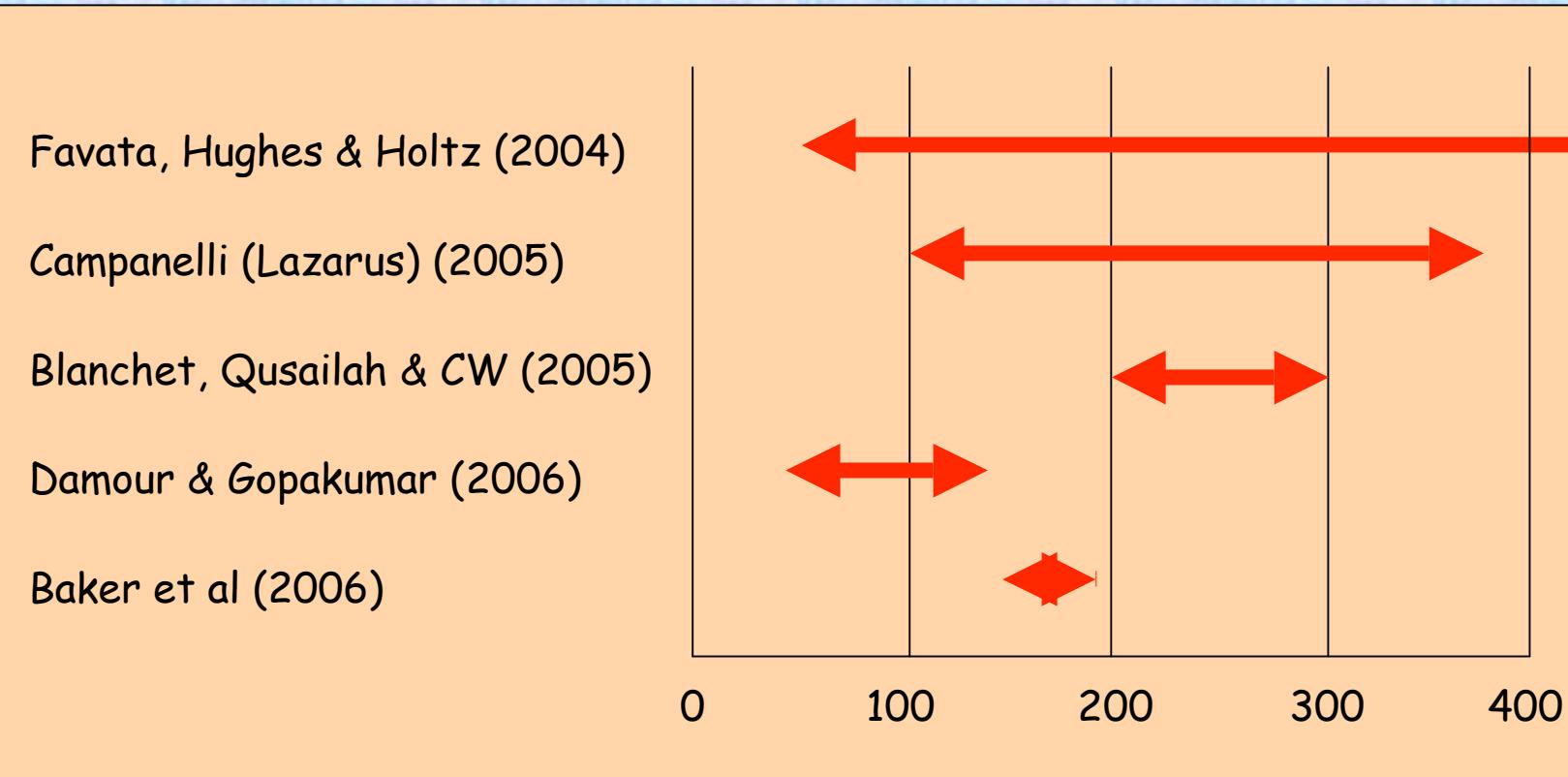


$$\eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$
$$= \frac{X}{(1 + X)^2}$$

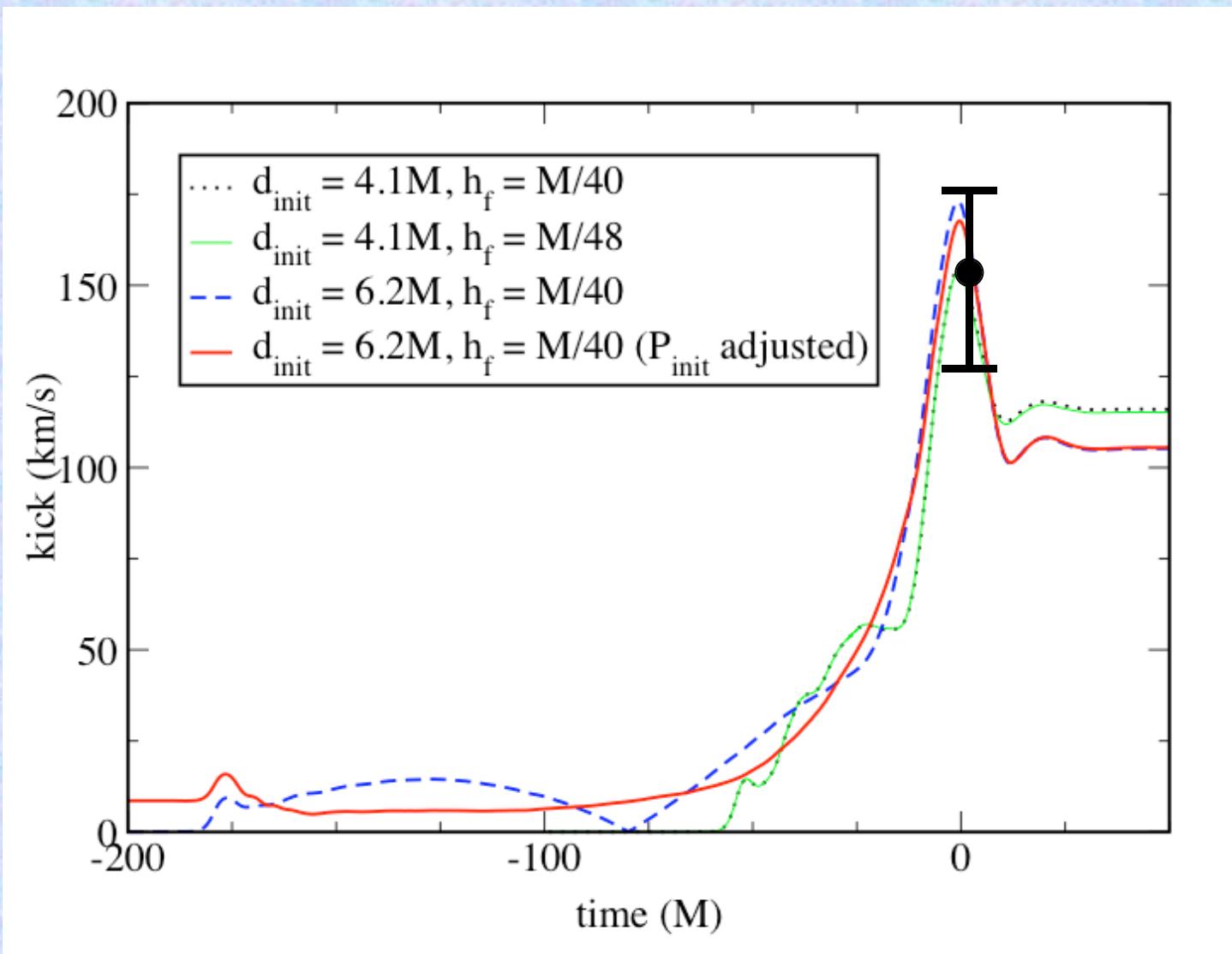
Blanchet, Qusailah & CW (2005)



Maximum recoil velocity: Range of Estimates



Getting a kick from numerical relativity



Baker *et al* (GSFC), gr-qc/0603204



The end-game of gravitational radiation reaction

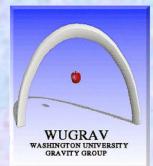
- Evolution leaves quasicircular orbit
 - describable by PN approximation
- Numerical models start with quasi-equilibrium (QE) states
 - helical Killing vector $\partial/\partial t + \Omega\partial/\partial\phi$
 - stationary in rotating frame
 - arbitrary rotation states (corotation, irrotational)
- How well do PN and QE agree for $E(\Omega)$, $J(\Omega)$?
 - “unreasonably” well, but some systematic differences exist
- Develop a PN diagnostic for numerical relativity
 - elucidate physical content of numerical models
 - “steer” numerical models toward more realistic physics

T. Mora & CMW, PRD 66, 101501 (2002) (gr-qc/0208089)

T. Mora & CMW, PRD 69, 104021 (2004) (gr-qc/0312082)

E. Berti, S. Iyer & CMW, PRD 74, 061503 (2006) (gr-qc/0607047)

E. Berti, S. Iyer & CMW (arXiv 0709.2589)



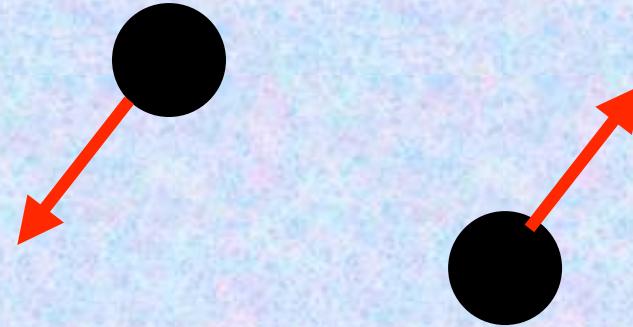
The PN-NR Interface - NR initial data

PN

- Evolution leaves quasicircular orbit
- Calculate $E(\Omega)$, $J(\Omega)$

NR

- Solve initial value equations of GR
- Calculate $E(\Omega)$, $J(\Omega)$



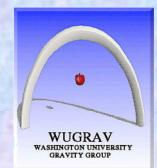
- How well do PN and NR agree for $E(\Omega)$, $J(\Omega)$?
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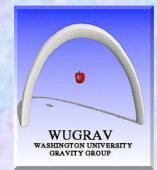
Ingredients of a PN Diagnostic

- Binding energy E & angular momentum J vs Ω
- Point-mass contributions to 3PN order

$$E \sim E_N \left\{ 1 + \frac{m}{r} + \left(\frac{m}{r} \right)^2 + \left(\frac{m}{r} \right)^3 \right\} \quad J \sim J_N \left\{ 1 + \frac{m}{r} + \left(\frac{m}{r} \right)^2 + \left(\frac{m}{r} \right)^3 \right\}$$

- Finite-size effects

- Rotational kinetic energy (2PN) $E_{Rot} \approx mR^2\omega^2 \approx E_N q^2(m/r)^2$
- Rotational flattening (5PN) $E_{Flat} \approx \delta I \omega^2 \approx \omega^4 R^5 \approx E_N q^5(m/r)^5$
- Tidal deformations (5PN) $E_{Tide} \approx (\delta' m)^2 / R \approx E_N q^5(m/r)^5$
- Spin-orbit (3PN) $E_{SO} \approx LS / r^3 \approx E_N q^2(m/r)^3$
- Spin-spin (5PN) $E_{SS} \approx S_1 S_2 / r^3 \approx E_N q^4(m/r)^5$



"Eccentric" orbits in relativistic systems

Define "measurable" eccentricity and semilatus rectum:

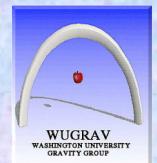
$$e \equiv \frac{\sqrt{\Omega_p} - \sqrt{\Omega_a}}{\sqrt{\Omega_p} + \sqrt{\Omega_a}}$$
$$\xi \equiv \frac{m}{p} \equiv \left(\frac{\sqrt{m\Omega_p} + \sqrt{m\Omega_a}}{2} \right)^{4/3} = \left(\frac{m\Omega_a}{(1-e)^2} \right)^{2/3}$$

Plusses:

- Exact in Newtonian limit
- Constants of the motion in absence of radiation reaction
- Connection to "measurable" quantities (Ω at infinity)
- "Easy" to extract from numerical data
- $e \rightarrow 0$ naturally under radiation reaction

Minuses

- Non-local
- Gauge invariant only through 1PN order



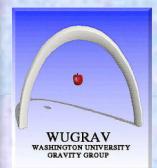
3PN Energy and Angular Momentum of "Eccentric" orbits

$$e \equiv \frac{\sqrt{\Omega_p} - \sqrt{\Omega_a}}{\sqrt{\Omega_p} + \sqrt{\Omega_a}}$$

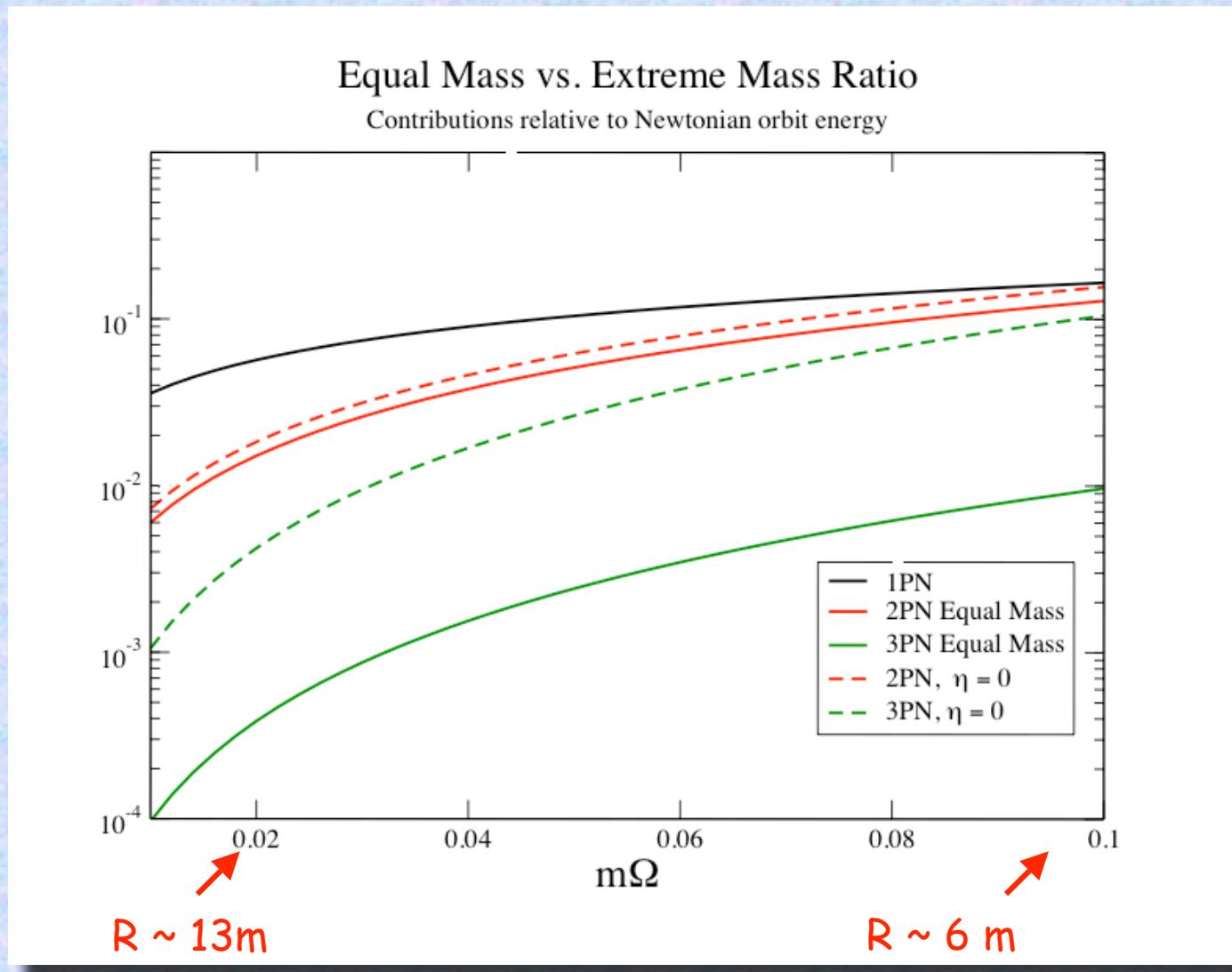
$$\xi \equiv \frac{m}{p} \equiv \left(\frac{\sqrt{m\Omega_p} + \sqrt{m\Omega_a}}{2} \right)^{4/3} = \left(\frac{m\Omega_a}{(1-e)^2} \right)^{2/3}$$

$$\begin{aligned} \tilde{E}_{\text{ADM}} &= -\frac{1}{2}(1-e^2)\zeta \left\{ 1 - \left[\frac{3}{4} + \frac{1}{12}\eta - \left(\frac{1}{12} - \frac{1}{4}\eta \right) e^2 \right] \zeta \right. \\ &\quad - \left[\frac{27}{8} - \frac{19}{8}\eta + \frac{1}{24}\eta^2 - \left(\frac{17}{12} + \frac{7}{4}\eta + \frac{1}{4}\eta^2 \right) e^2 \right. \\ &\quad \left. + \left(\frac{1}{24} + \frac{11}{24}\eta - \frac{1}{8}\eta^2 \right) e^4 \right] \zeta^2 \\ &\quad - \left[\frac{675}{64} - \left(\frac{34445}{576} - \frac{205}{96}\pi^2 \right) \eta + \frac{155}{96}\eta^2 + \frac{35}{5184}\eta^3 \right. \\ &\quad \left. + \left(\frac{7}{64} + \left(\frac{167}{64} - \frac{41}{96}\pi^2 \right) \eta + \frac{7595}{864}\eta^2 - \frac{25}{576}\eta^3 \right) e^2 \right. \\ &\quad \left. - \left(\frac{815}{576} - \frac{6995}{1728}\eta - \frac{299}{288}\eta^2 - \frac{25}{64}\eta^3 \right) e^4 \right. \\ &\quad \left. - \left(\frac{35}{5184} - \frac{31}{192}\eta + \frac{13}{32}\eta^2 - \frac{5}{64}\eta^3 \right) e^6 \right] \zeta^3 \Big\}, \end{aligned}$$

$$\begin{aligned} \tilde{J}_{\text{ADM}} &= \frac{1}{\sqrt{\zeta}} \left\{ 1 + \left[\frac{3}{2} + \frac{1}{6}\eta - \left(\frac{1}{6} - \frac{1}{2}\eta \right) e^2 \right] \zeta \right. \\ &\quad + \left[\frac{27}{8} - \frac{19}{8}\eta + \frac{1}{24}\eta^2 + \left(\frac{23}{12} - \frac{35}{12}\eta - \frac{1}{4}\eta^2 \right) e^2 \right. \\ &\quad \left. + \left(\frac{1}{24} - \frac{17}{24}\eta - \frac{1}{8}\eta^2 \right) e^4 \right] \zeta^2 \\ &\quad + \left[\frac{135}{16} - \left(\frac{6889}{144} - \frac{41}{24}\pi^2 \right) \eta + \frac{31}{24}\eta^2 + \frac{7}{1296}\eta^3 \right. \\ &\quad \left. + \left(\frac{299}{16} - \left(\frac{1025}{16} - \frac{41}{24}\pi^2 \right) \eta + \frac{2077}{216}\eta^2 - \frac{5}{144}\eta^3 \right) e^2 \right. \\ &\quad \left. + \left(\frac{77}{144} - \frac{1337}{432}\eta + \frac{271}{72}\eta^2 + \frac{5}{16}\eta^3 \right) e^4 \right. \\ &\quad \left. - \left(\frac{7}{1296} - \frac{7}{48}\eta + \frac{3}{8}\eta^2 - \frac{1}{16}\eta^3 \right) e^6 \right] \zeta^3 \Big\}. \end{aligned}$$



How well does the PN expansion converge?



Tidal and rotational effects

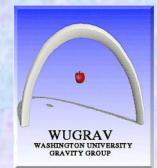
- Use Newtonian theory; add to 3PN & Spin results
 - standard textbook machinery (eg Kopal 1959, 1978)
 - multipole expansion -- keep $l=2$ & 3
 - direct contributions to E and J
 - indirect contributions via orbit perturbations
- Dependence on 4 parameters

$$\alpha = I / MR^2$$

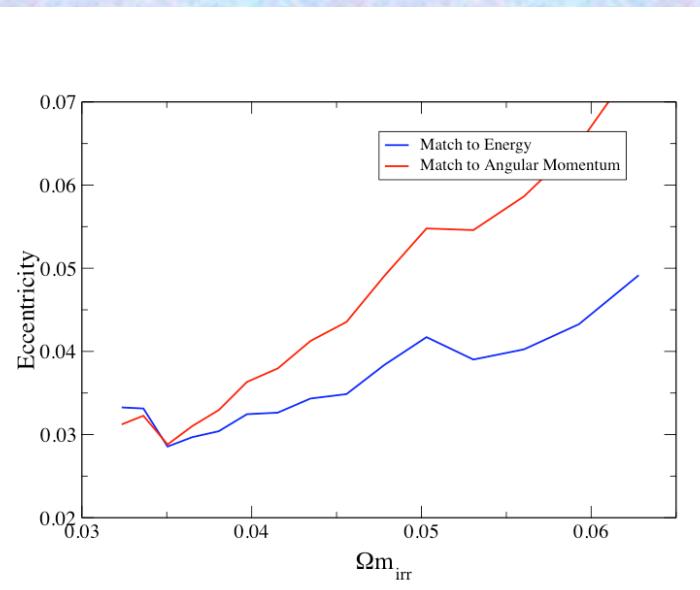
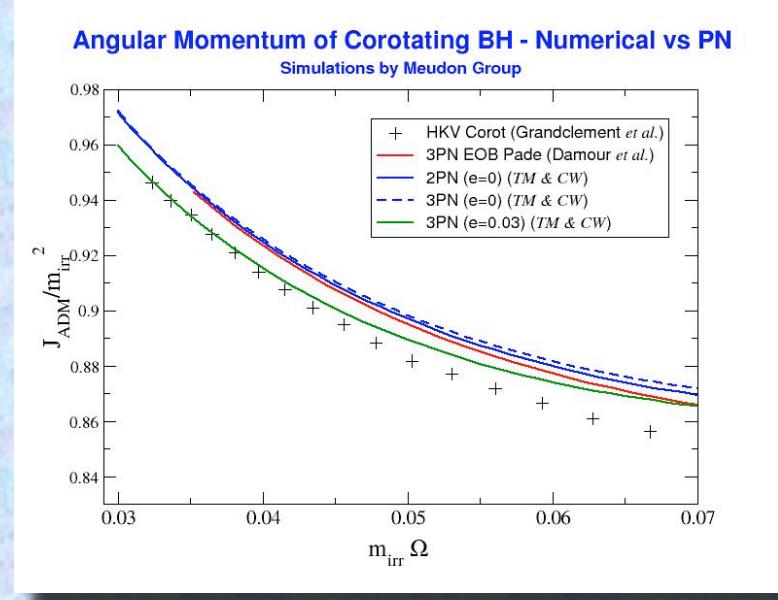
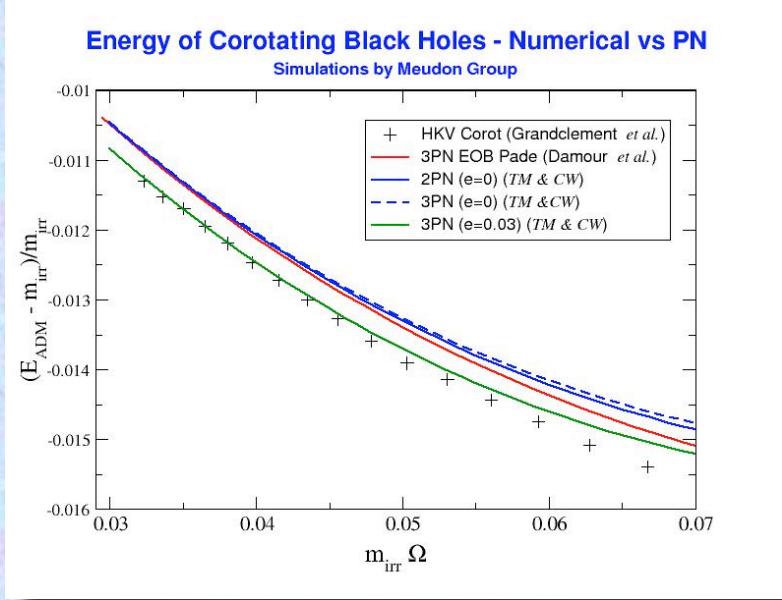
$$q = R / M$$

k_2, k_3 : apsidal constants

$$k_l = \begin{cases} 0 & , \text{ point mass} \\ \frac{3}{4(l-1)} & , \text{ homogeneous} \end{cases}$$

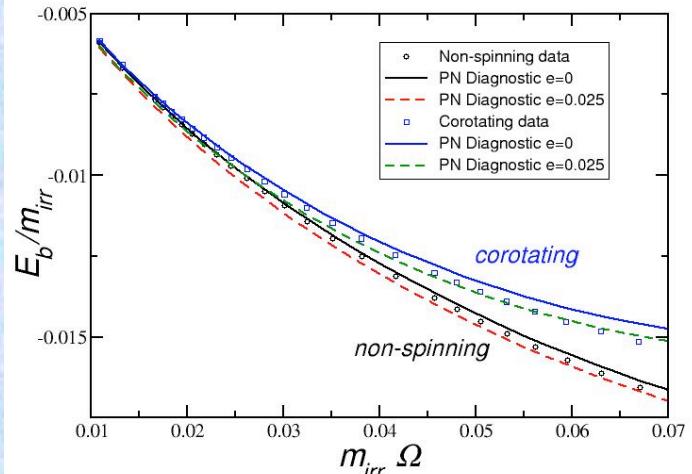


Corotating Black Holes - Meudon Data

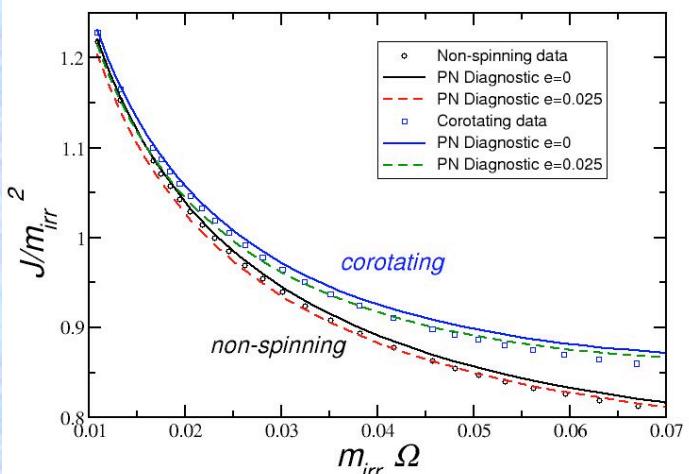


Corotating and Non-Spinning Binary Black Holes

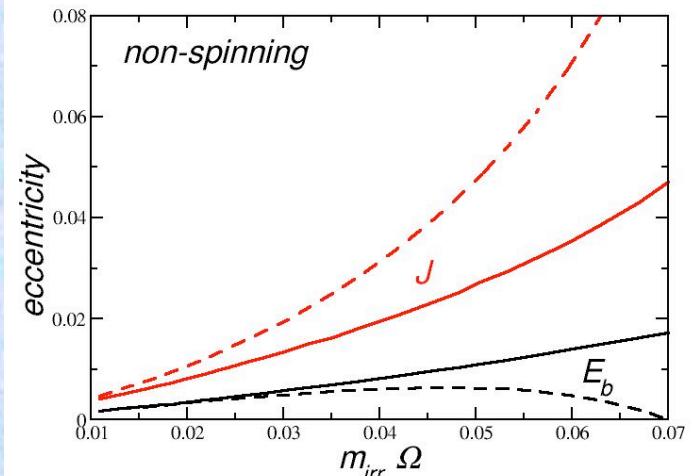
Caudill et al



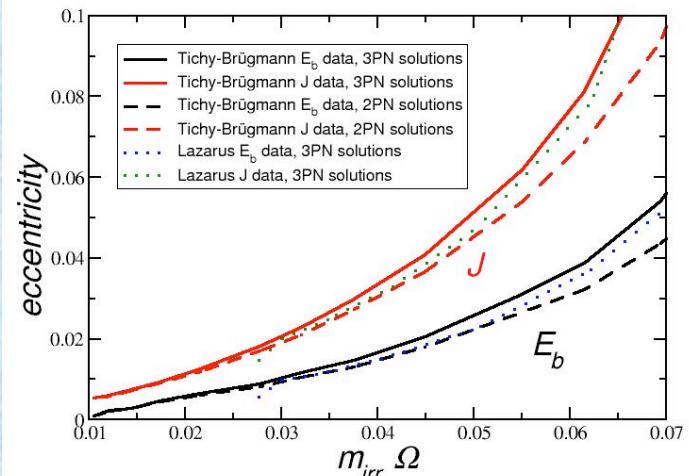
Caudill et al



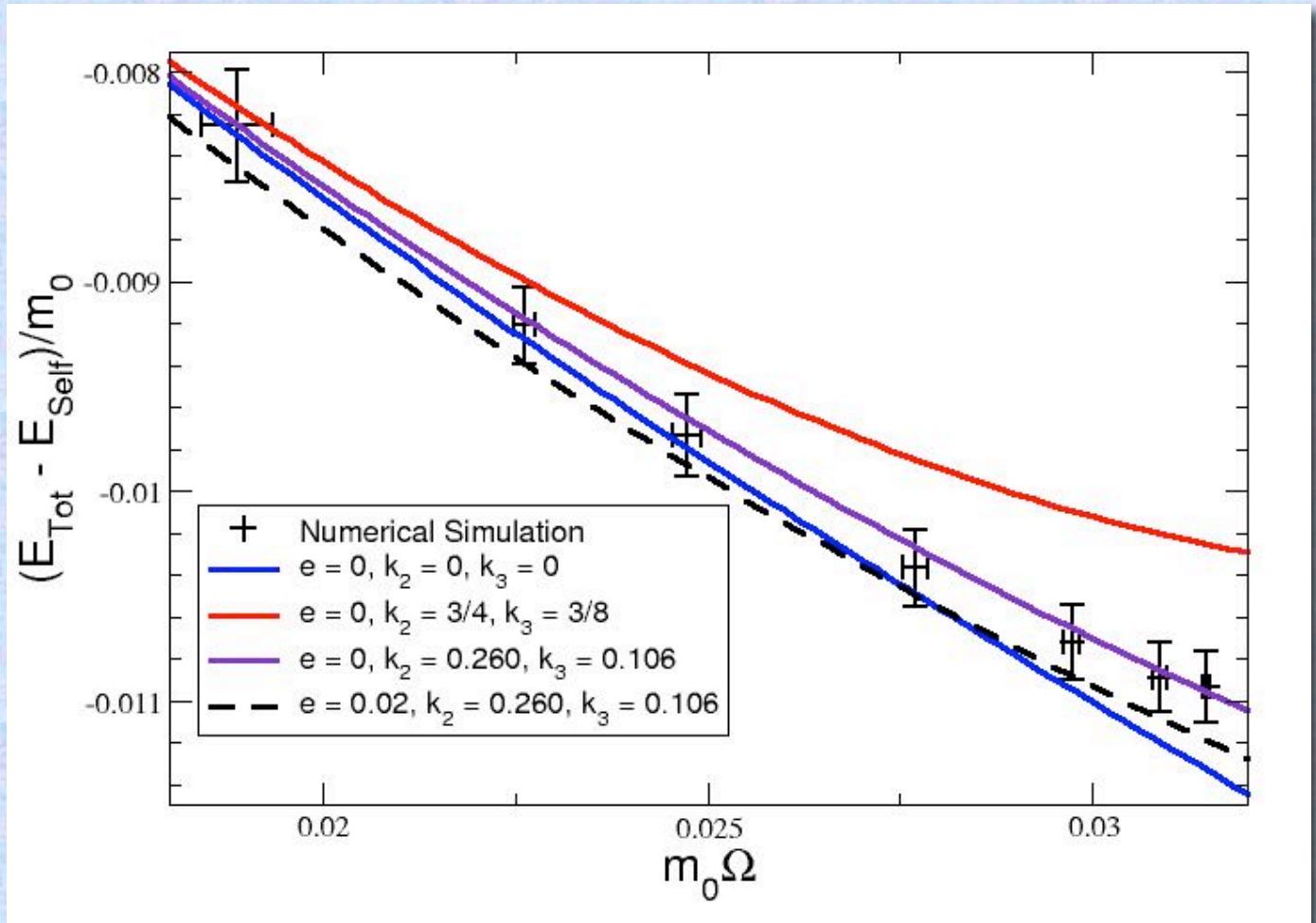
Cook-Pfeiffer, Caudill et al



Tichy-Brügmann, Lazarus



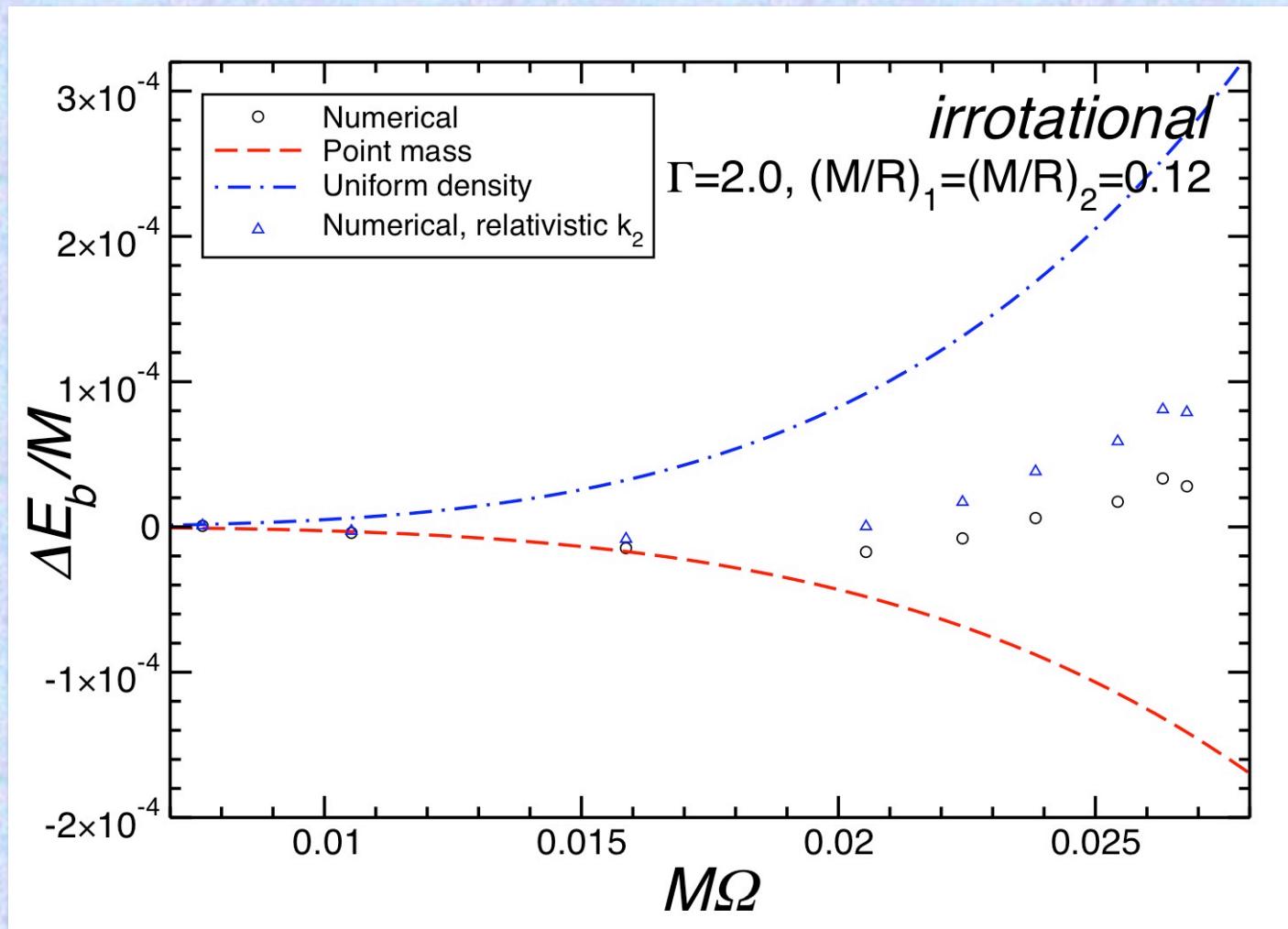
Energy of Corotating Neutron Stars - Numerical vs. PN



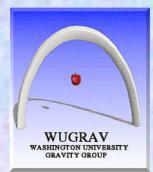
Simulations by Miller, Suen & WUGRAV



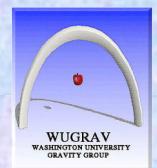
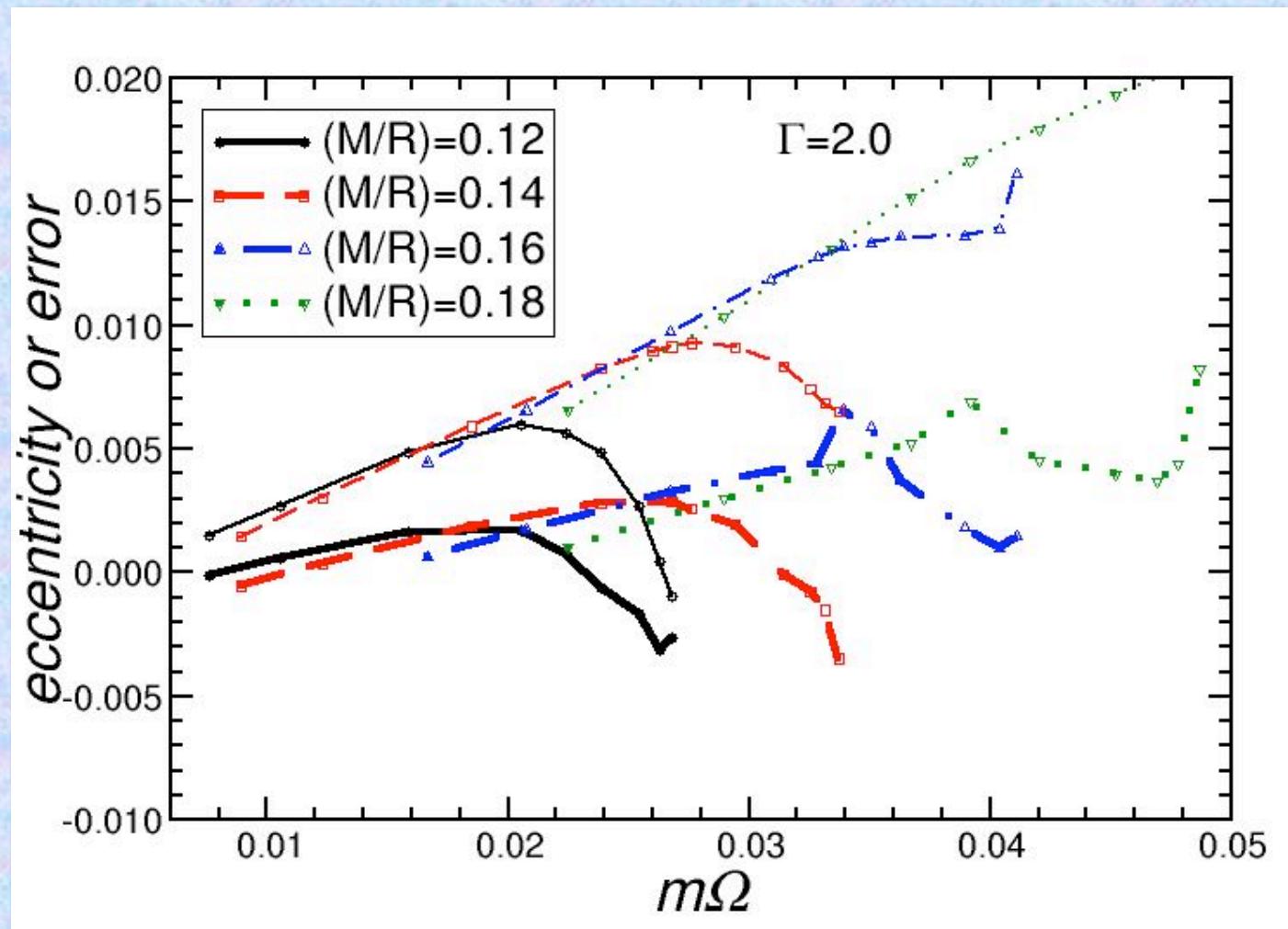
Energy of irrotational neutron stars - PN vs Meudon/Tokyo



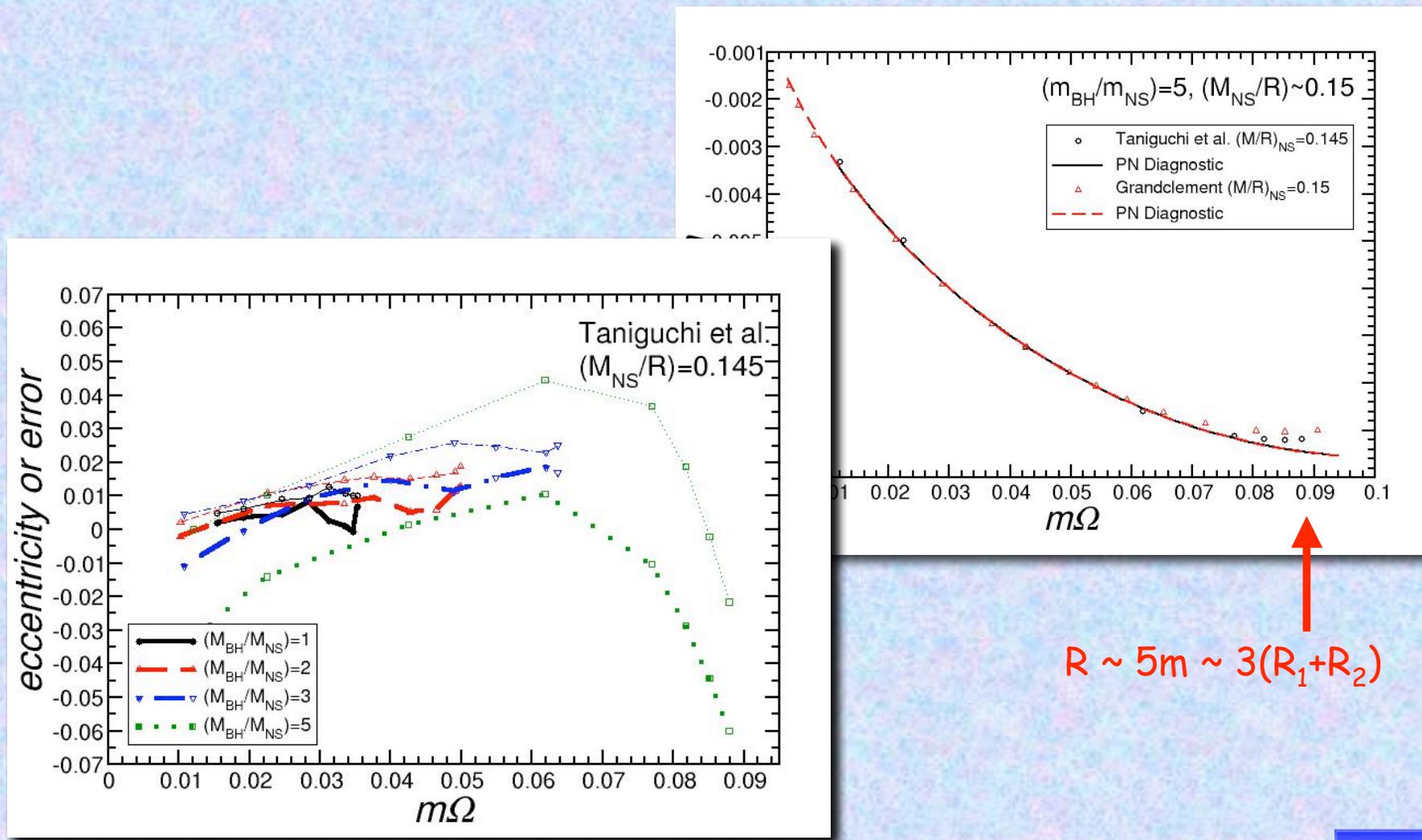
Data from Taniguchi & Gourgoulhon PRD 68, 124025 (2003)
Diagnostic by Berti, Iyer & CMW (2007)



Inferred eccentricities: irrotational neutron stars

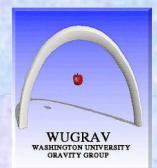


BH - NS Initial Configurations



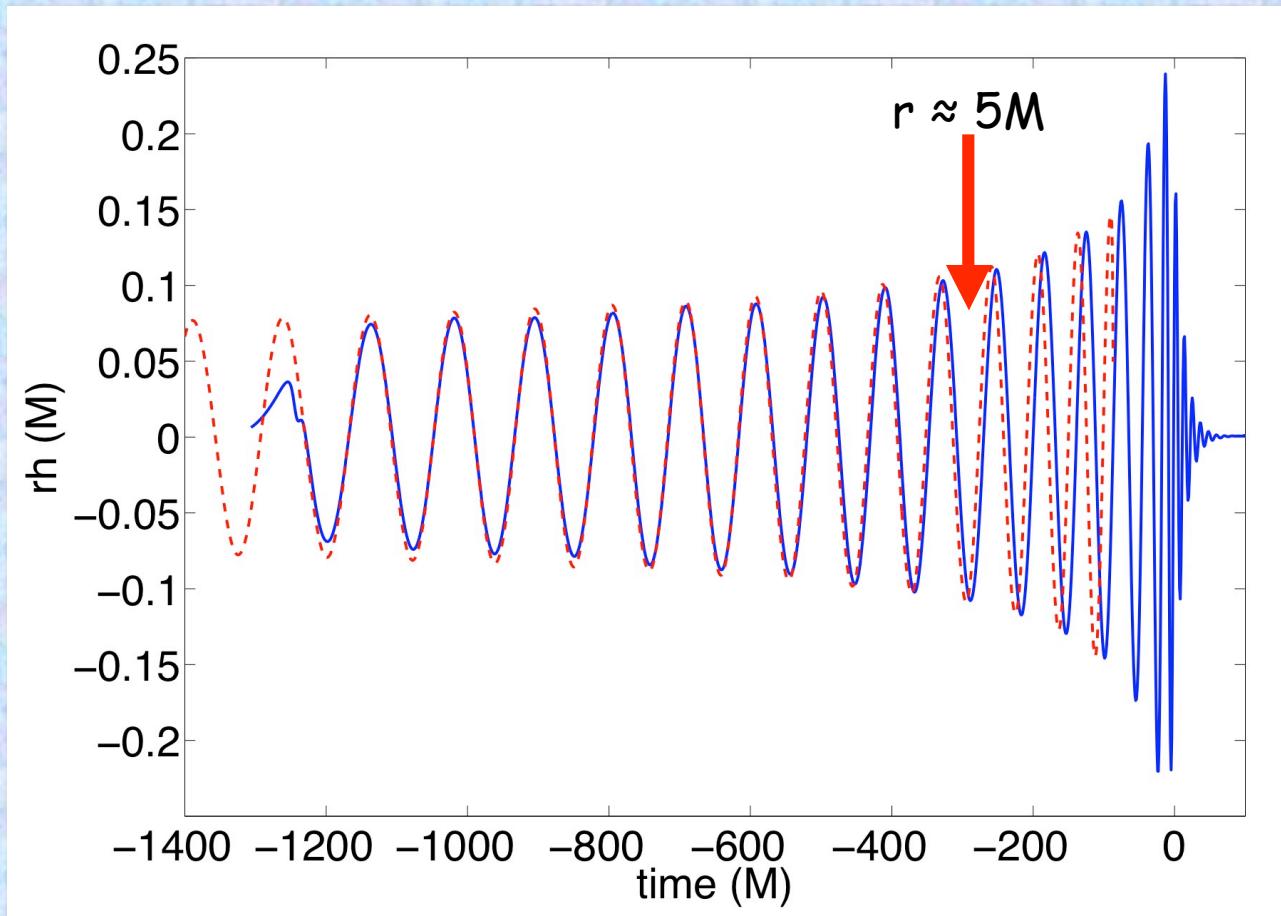
$$R \sim 5m \sim 3(R_1 + R_2)$$

Data from Taniguchi et al, gr-qc/0701110
 Grandclement, gr-qc/0609044 [v5]



NRm3PN

Comparing High-order PN with Numerical Waveforms

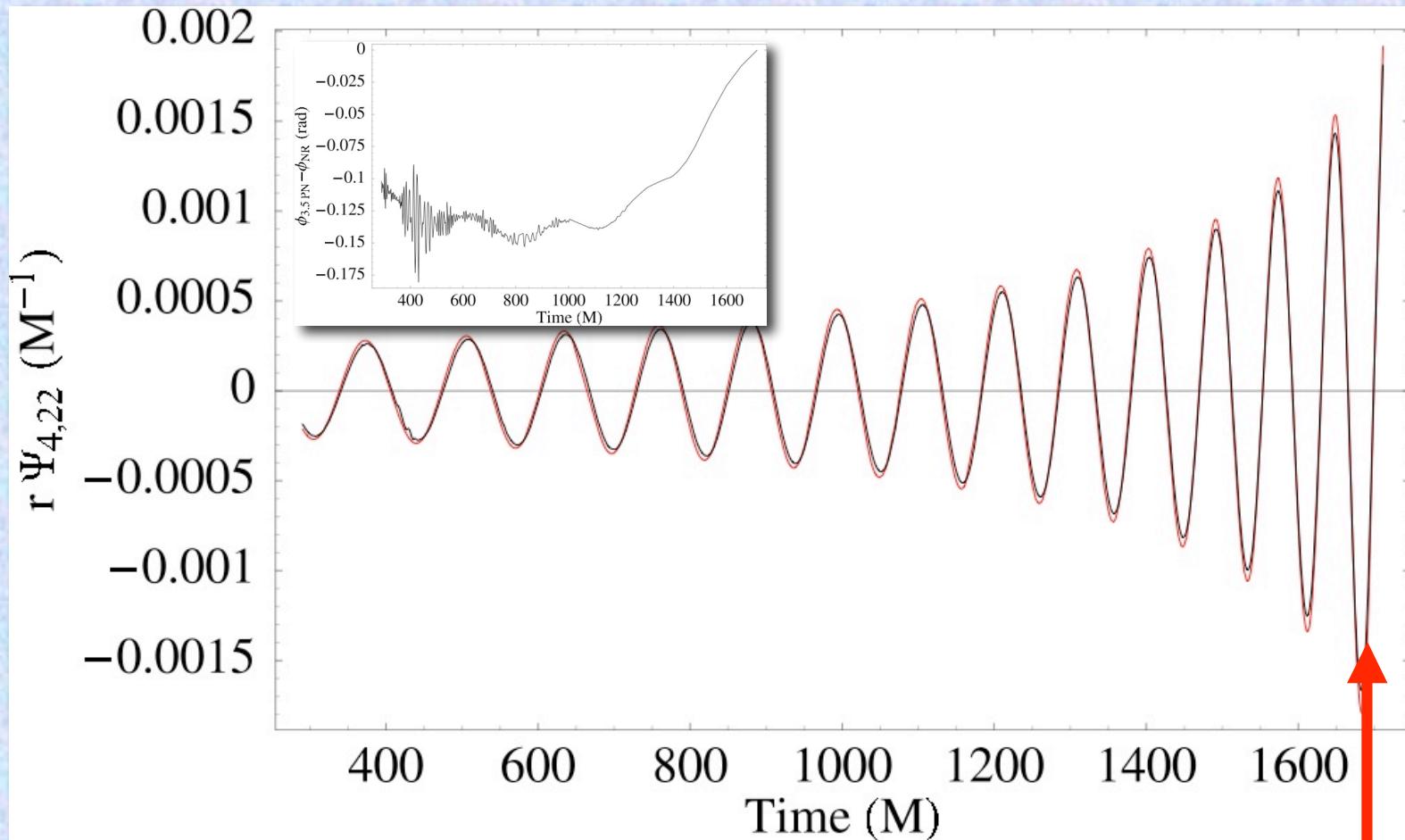


Baker *et al.* gr-qc/0612024



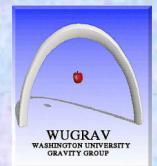
NRm3PN

Comparing High-order PN with Numerical Waveforms



Hannam *et al.* arXiv:0706.1305
Boyle *et al.* arXiv:0710.0158

$r \approx 4.6M$



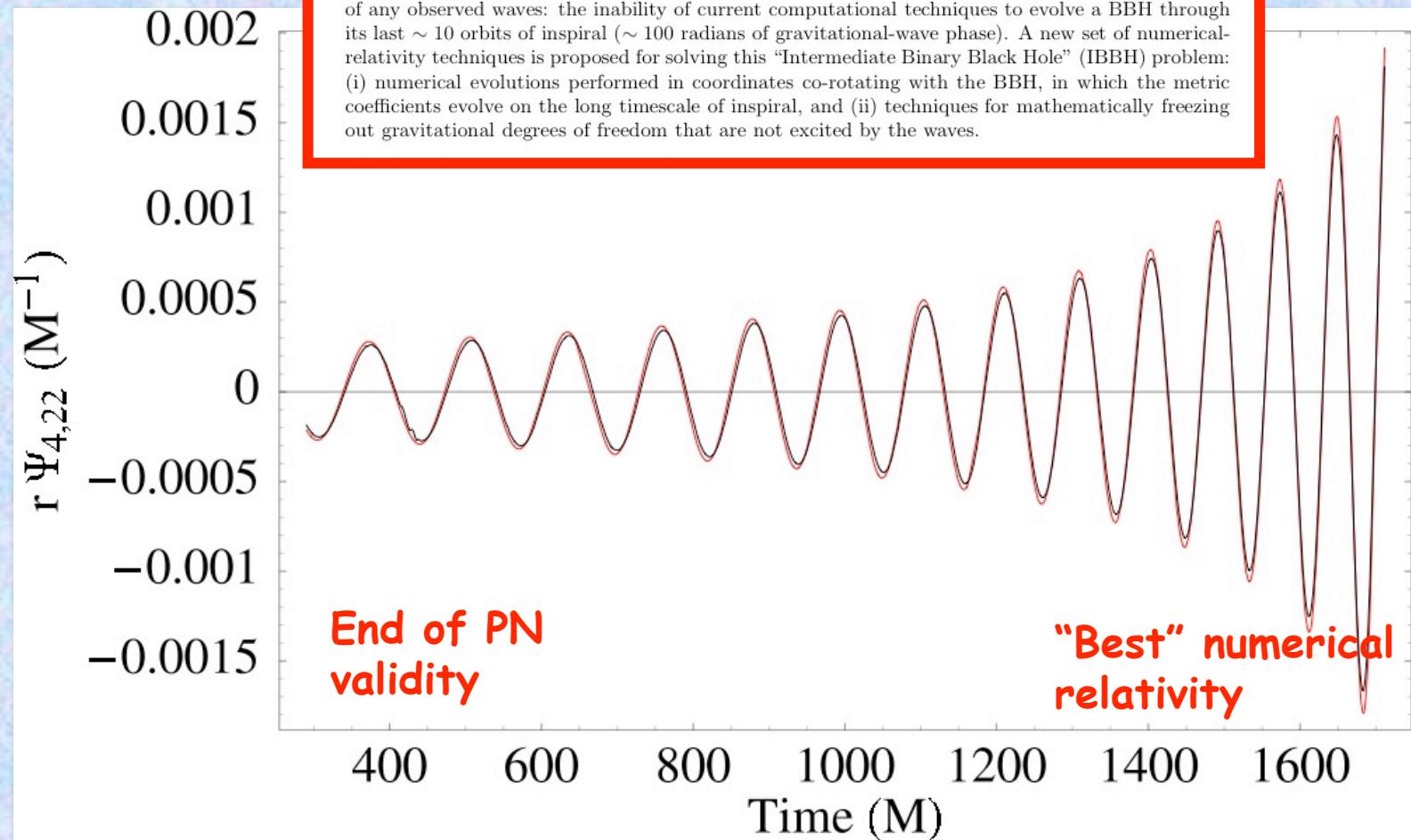
Reasonable vs. Unreasonable

Computing the merger of black-hole binaries: the IBBH problem

Patrick R. Brady, Jolien D. E. Creighton, and Kip S. Thorne

*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125
(22 April 1998)*

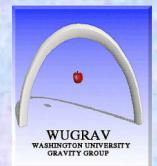
Gravitational radiation arising from the inspiral and merger of binary black holes (BBH's) is a promising candidate for detection by kilometer-scale interferometric gravitational wave observatories. This paper discusses a serious obstacle to searches for such radiation and to the interpretation of any observed waves: the inability of current computational techniques to evolve a BBH through its last ~ 10 orbits of inspiral (~ 100 radians of gravitational-wave phase). A new set of numerical-relativity techniques is proposed for solving this "Intermediate Binary Black Hole" (IBBH) problem: (i) numerical evolutions performed in coordinates co-rotating with the BBH, in which the metric coefficients evolve on the long timescale of inspiral, and (ii) techniques for mathematically freezing out gravitational degrees of freedom that are not excited by the waves.



NRm3PN

Current Issues

- How to stitch together PN and NR waveforms
- How much S/N is lost in various stitching schemes?
- What happened to the plunge?
- Can PN waveforms teach us about ringdown waves?
- How much of the kick is PN, how much is ringdown braking?
- How does spin change the picture?



On the unreasonable effectiveness of post-Newtonian theory in gravitational-wave physics

- Introduction
- The problem of motion & radiation - a history
- Post-Newtonian theory
- “Unreasonable accuracy”
 - Binary pulsars
 - Gravitational-wave kicks
 - Initial data for binary inspiral
 - PN-numerical waveform matching
- Epilogue



Clifford Will, FermiLab Colloquium, 9 Jan 2008